The Spin Rate of a Ping Pong Ball and the Deflection Ratio

EXPLORATION & PERSONAL ENGAGEMENT

Introduction

I have played sports all my life. Soccer is my passion. I play it, I watch it, I love it, the beautiful game! To me, there is nothing better than when a penalty kick curves around the wall of defenders and into the corner of the goal. As a child, I was always curious as to why that works. I was interested to know, "What is the physics behind a curving soccer ball?" Now that I'm in IB Physics, I know enough to be able to finally learn about this topic for myself. So, for this investigation, I decided to investigate the physics of a curving soccer ball.

As a Ping-Pong player, I know that you can curve a Ping-Pong ball even more sharply than you can curve a soccer ball. It's also quicker and easier to work with hitting a Ping-Pong ball in the limited space of our school lab. So I decided to experiment with a curving Ping-Pong ball instead of with a soccer ball, since the physics is the same for both.

Comment [1]: **The evidence of personal engagement with the exploration is clear with significant independent thinking, initiative or creativity.**

The justification given for choosing the research question and/or the topic under investigation demonstrates **personal significance**, **interest or curiosity**.

Background Theory

When a Ping-Pong ball is hit with a paddle, the amount of spin that is put onto the ball affects the amount of curvature it experiences during its flight. The part of the ball spinning in the direction of the air flow will cause the air on that side to travel faster, allowing the air to travel faster as well, while on the other side, the oppositely spinning area will cause that portion of the air to travel slower, causing that portion of the air to travel less quickly. The air will exert a force on the side of the ball spinning in the direction of the air and due to Newton's third law of motion the ball will exert an equal force towards the air, which is known as the Magnus Force. Ultimately the Magnus Force causes the ball to bend towards a certain direction, depending on the spin. (http://ffden-phys.uaf.edu/211_fall2010.web.dir/Patrick_Brandon/what_is_the_magnus_effect.html) Assuming that the Ping-Pong and the environment are kept constant, the two main factors affecting the Magnus Force produced is the spin rate, or angular frequency, of the ball and the velocity at which the ball travels. This investigation serves to study how the spin rate of a Ping-Pong ball affects the amount of curvature produced.

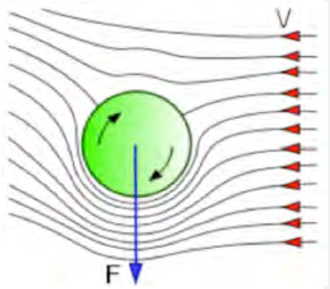

Figure 1: Graphical representation of the Magnus Effect. Clearly, the spin is causing a perpendicular force, which causes the ball to curve.

Comment [2]: The background information provided for the investigation is entirely appropriate and relevant and enhances the understanding of the context of the investigation.

There is evidence of **personal input and initiative** in the designing, implementation or presentation of the investigation

Research Question: How does the spin rate of a Ping-Pong ball affect its deflection ratio (defined below)?

Comment [3]: The topic of the investigation is identified and a relevant and fully focused research question is clearly described.

According to the Journal of Physics of the University of Leicester (https://physics.le.ac.uk/journals/index.php/pst/article/view/458/256), the effect of a spherical object's spin rate on the amount of curvature it undergoes can be modeled by the following equation[i]:

$$D = \frac{\pi R^3 \rho \omega}{v m} x^2 \quad \text{[Equation 1]}$$

Comment [4]: **The evidence of personal engagement with the exploration is clear with significant independent thinking, initiative or creativity.**

There is evidence of **personal input and initiative** in the designing, implementation or presentation of the investigation

where D is the distance the ball travelled in the direction of the Magnus Force, R is the radius of the Ping-Pong ball, ρ is the density of the air in the room, ω is the spin rate, v is the velocity of the ball, m is the mass of the ball, and x is the distance travelled. The equation was specifically designed to fit the motion of soccer balls, but it fit the motion of Ping-Pong balls equally accurately since the assumptions set forth applied for smaller sized spherical objects. Equation 1, however, called for three independent variables, ω, v, and x; therefore another equation was derived to keep all the values except ω and D constant. Since velocity can be expressed as $v = x/t$, t was substituted into Equation 1 and an equation with one independent variable, the spin rate, was derived.

$$\frac{D}{x} = \frac{\pi R^3 \rho t}{m}\omega \quad \text{[Equation 2]}$$

Comment [5]: **The evidence of personal engagement with the exploration is clear with significant independent thinking, initiative or creativity.**

There is evidence of **personal input and initiative** in the designing, implementation or presentation of the investigation

Reducing the number of independent variables did not show a direct relationship between spin rate and the horizontal curvature, D, but instead with the value of $\frac{D}{x}$, the deflection ratio, defined as the amount of curvature in the direction of the Magnus Force set as the y-axis over the total distance travelled in the x-axis.

The final derived equation shows a direct relationship between the spin rate of the Ping-Pong ball and the deflection ratio, $\frac{D}{x}$. The relationship best fits the form of a proportional fit, $y = Ax$; therefore, $\frac{D}{x}$ is predicted to be proportional to ω with a proportionality constant of $\frac{\pi R^3 \rho t}{m}$. The final graph is predicted to be a $\frac{D}{x}$ versus ω proportional graph with a gradient of $\frac{\pi R^3 \rho t}{m}$.

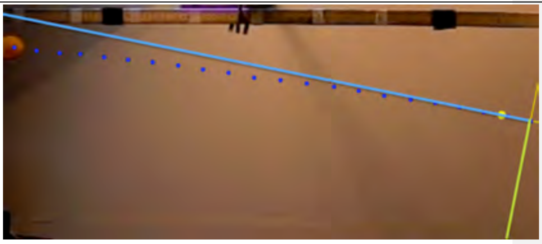

Figure 2: The blue axis was used to determine, D, the amount the ball travelled in the direction of the Magnus Force and the green axis was used to determine, x, the distance travelled in the direction of the initial hit. The two values were written as ratios of $\frac{D}{x}$, which ultimately was defined as the deflection ratio of the ball.

Comment [6]: The background information provided for the investigation is entirely appropriate and relevant and enhances the understanding of the context of the investigation

Methods

Variables
Independent Variable: Spin rate of the Ping-Pong ball
Dependent Variable: Deflection Ratio
Controlled Variable(s):

- Ping-Pong Ball: The same Ping-Pong ball was used throughout the experiment in order to maintain consistency in the data. A "Changyun 1 star" Ping-Pong ball with a mass of 2.36 ± 0.01 grams and a diameter of 4.00 ± 0.01 cm was used.
- High Speed Camera: The same camera was used in recording all the videos in order to make sure that all the footage came from the same source and was kept consistent. A Casio EX-F1 camera was used and set at 600 frames per second (fps) as this provided enough quality to analyze the videos.
- Temperature of Environment: The temperature was controlled by staying in the same area of the room throughout the experiment. The temperature of the room was 25 ± 1°C.
- Elapsed Time: Since Equation 3 required time to be a constant value, the same number of frames were analyzed in all the trials. Exactly 105 frames were analyzed for all the trials in order to keep time constant.

Safety, Ethical, or Environmental Issues

Reasonable safety precautions were taken during the setup and conduct of the experiment. The high speed camera was firmly secured in position so it could not fall and break or injure the ping-pong hitter. The hot spotlights were placed far enough away from the ping-pong hitter to ensure that he did not accidentally touch them. Fragile items (glassware) were removed from the area the ping-pong was being hit towards so it wouldn't knock them over.

There were no identified ethical or environmental issues that needed to be addressed in this investigation.

Comment [7]: The report shows evidence of full awareness of the significant safety, ethical or environmental issues that are **relevant to the methodology of the investigation**

Procedure

First, in order to detect the full motion of the Ping-Pong ball accurately, the camera was placed over the Ping-Pong table to view the system in a 'bird's eye view'. The camera was connected to a tripod and then clamped to a cabinet shelf using metal clamps in order to get enough distance between the table and the camera. The high-speed feature on the camera required abundant space due to its limited field of vision. To increase the quality of the videos, two spotlights were placed beside the table. Before the hits were recorded, a line was drawn around the center of the ball where the two semi-spheres of the ball came together and formed a narrow ridge. Then a short line perpendicular to the central line was drawn in order to act as a guide in detecting the ball's spins during the video analysis process. The spotlights were turned on and at first the Ping-Pong ball was hit with very little, or almost no spin at all. Since it was impossible to have three trials with the same amount of spin, 40 trials were recorded and the amount of spin added was increased every time alternating between left and right spin.

Figure 3: This was the set up of the experiment. The camera was set up on top and a ruler was set up in order to keep the hits around its height. Lighting was used to increase the quality of the videos and the Ping-Pong ball was hit with a wide variety of sidespins, from 0 to 164 rad/s.

After all the hits were recorded, 20 good trials, ranging from no spin to maximum spin, were picked out and analyzed. First, the spin rate was measured by seeing how long it took the ball to spin horizontally a certain number of times. Then the central point of the ball was tracked for exactly 105 frames at a five-frame interval. Afterwards, the origin of the graph was set on the first data point which gave two sets of data points, one representing ball's movement in the direction of the Magnus Force and the other the movement in the direction the ball was hit. A quadratic equation was used to best fit the former and a linear equation for the latter. Using the equations, the final and initial times were substituted into the quadratic equation and subtracted from each other to find the value of D, and the same process was repeated with the linear equation to find the value of x. Then the absolute value of the difference was used as the final values of D and x since only the magnitude of the movements was needed.

Comment [8]: The methodology of the investigation is highly appropriate to address the research question because it takes into consideration all, or nearly all, of the significant factors that may influence the relevance, reliability and sufficiency of the collected data.

ANALYSIS

Data

Spin Rate (±3 rad/s)	Graph of Y			Graph of X		Time	
	A	B	C	m	b	Initial	Final
0	0.0298	-11.94	1195	108.5	-21200	195.5	199.0
11	0.1246	-55.83	6254	138.0	-31240	226.3	229.8
14	-0.5285	131.8	-8213	140.8	-17250	122.5	126.0
29	-0.7679	250.9	-20480	141.3	-22730	160.9	164.4
72	0.2720	-1130	11730	134.0	-28030	209.1	212.6
97	1.528	-1589	413000	119.0	-61810	519.6	523.1
99	1.373	-1554	439900	138.4	-78390	566.3	569.8
101	0.7347	-1165	461900	158.9	-126500	796.0	799.5
102	0.9991	-483.1	58390	136.2	-33090	242.8	246.3
106	1.815	-1467	296400	225.0	-91140	405.1	408.6
108	1.768	-507.9	36480	152.6	-21990	144.0	147.5
109	0.9766	-1036	274900	154.0	-82080	532.8	536.3
111	1.723	-1351	264900	116.3	-45600	392.1	395.6
114	1.962	-2181	605900	128.1	-71180	555.8	559.3
118	4.704	-3800	767500	205.0	-82670	403.3	406.8
130	2.034	-2929	1054000	180.8	-130300	720.8	724.3
135	0.9491	-1294	441100	228.2	-156500	685.8	689.3
140	1.666	-380.8	21770	134.5	-15450	114.9	118.4
151	2.677	-1189	132100	135.1	-29990	222.0	225.5
164	1.900	-1940	495400	199.8	-102300	512.1	515.6

Table 1: The raw data table of the collected values all rounded to four significant figures. As seen above time was a constant variable in the experiment; however, LoggerPro was claiming that one second was equivalent to 30 frames, which was not the case. Therefore, the time range, 3.5 seconds was multiplied by a factor of $\frac{30\ frames}{600\ frames}$(simplified to $\frac{1}{20}$), since the camera was set at 600 frames per second. After the calculation, the time range was found to be 0.175 ± 0.001 seconds. Analyzing one trial three times and dividing the range by a factor of two achieved the uncertainty of the spin rate. This uncertainty was applied to all the other trials.

Data Processing

Sample Calculations

1. **Spin Rate**

$$Spin\ Rate = \frac{2n\pi}{\frac{Range\ of\ frames\ for\ n\ revolutions}{600\ frames}}$$

where n is the number of revolutions the ball spun within a specific number of frames.

$$= \frac{2\pi}{\frac{3544 - 3517}{600}}$$

$$= 139.625 = 137\frac{rad}{s}$$

2. Spin Rate Uncertainty

$$Uncertainty = \frac{Maximum\ Value - Minimum\ Value}{2}$$

$$= \frac{144.997 - 139.626}{2}$$

$$= 2.6855 \quad = \pm\frac{3\ radians}{second}$$

3. Time Uncertainty

$$Uncertainty = \frac{Range}{2}$$

$$= \frac{\frac{1}{600}}{2}$$

$$= \frac{1}{1200}$$

$$= 0.000833 \quad = \pm 0.001\ s$$

Sample Graph

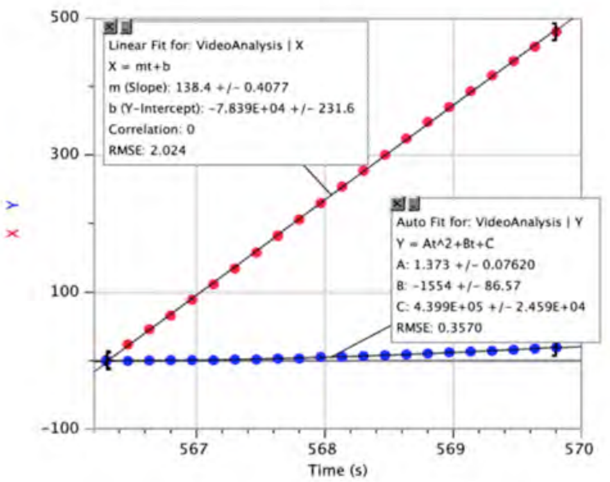

Figure 4: Sample graph showing the two fits made in order to obtain the values of *D* and *x*. The coefficient values shown in the graphs were used in determining the values as explained in the procedure.

Raw Data Table

Spin Rate (±3 rad/s)	D (±2 pixels)	x (±30 pixels)
0	0	380
11	4	480
14	2	490
29	4	490
72	6	470
97	15	420
99	21	480
101	25	560
102	19	480
106	35	790
108	26	530
109	28	540
111	22	410
114	24	450
118	27	720
130	36	630
135	39	800
140	28	470
151	31	470
164	44	700

Table 2: The values of D and x were left in pixels since they were going to be written as ratios and converting pixels into meters would ultimately give the same values. Uncertainties were calculated by analyzing one trial three times and dividing the range of answers by two.

Sample Calculations

4. *D* Value

$$D = \left(At_f^2 + Bt_f + C\right) - \left(At_i^2 + Bt_i + C\right)$$

$$= (1.666(118.4^2) - 380.8(118.4) + 21770) - (1.666(114.9^2) - 380.8(114.9) + 21770)$$

$$= 27.5723\ pixels$$

5. *x* Value

$$x = \left(mt_f + b\right) - (mt_i + b)$$

$$= (134.5(118.4) - 15450) - (134.5(114.9) - 15450)$$

$$= 470.75\ pixels$$

6. Uncertainty of D

$$Uncertainty = \frac{Maximum\ Value - Minimum\ Value}{2}$$

$$Maximum\ Value = (1.894(118.4^2) - 433.1(118.4) + 24750) - (1.894(114.9^2) - 433.1(114.9) + 24750)$$

$$Minimum\ Value = (1.666(118.4^2) - 380.8(118.4) + 21770) - (1.666(114.9^2) - 380.8(114.9) + 21770)$$

$$= \frac{30.6957 - 27.5723}{2}$$

$$= 1.5617 \quad = \pm 2$$

7. **Uncertainty of *x***

$$Uncertainty = \frac{Maximum\ Value - Minimum\ Value}{2}$$

$$Maximum\ Value = (151.8(118.4) + 17440) - 151.8(114.9) + 17440))$$

$$Minimum\ Value = ((134.5(118.4) + 15450) - (134.5(114.9) + 15450)$$

$$= \frac{531.3 - 470.8}{2}$$

$$= 30.275 \quad = \pm 30$$

Final Table

Spin Rate (±3 rad/s)	D/x (±0.02)	Spin Rate (±3 rad/s)	D/x (±0.02)
0	0.00	108	0.05
11	0.01	109	0.05
14	0.00	111	0.05
29	0.01	114	0.05
72	0.01	118	0.05
97	0.04	130	0.06
99	0.04	135	0.05
101	0.05	140	0.06
102	0.04	151	0.07
106	0.04	164	0.06

Table 3: Final table of the spin rate and $\frac{D}{x}$ values.

Sample Calculations

8. $\frac{D}{x}$ **value**

$$\frac{D}{x} = \frac{28}{471}$$

$$= 0.059448 \quad = 0.06$$

9. Uncertainty of $\frac{D}{x}$

$$Uncertainty = \frac{Maximum\ Value - Minimum\ Value}{2}$$

$$= \frac{\left(\frac{30}{440}\right) - \left(\frac{26}{800}\right)}{2}$$

Results

Final Graph

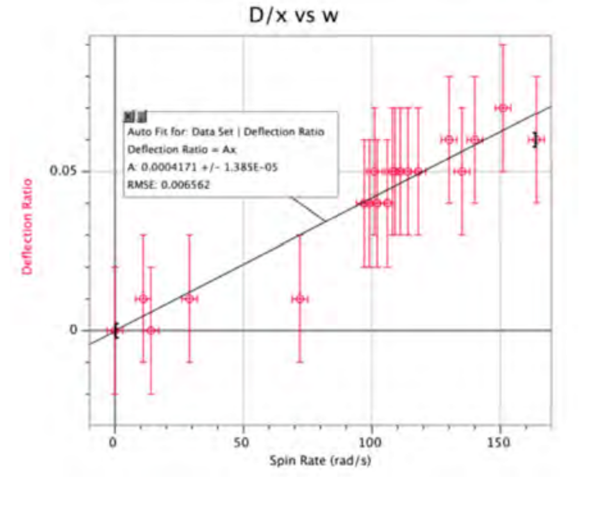

Figure 5: Final proportional graph of the relationship between spin rate and the deflection ratio.

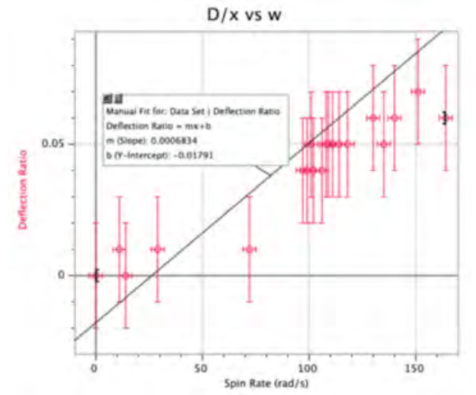

Figure 6: Linear graph of highest possible slope.

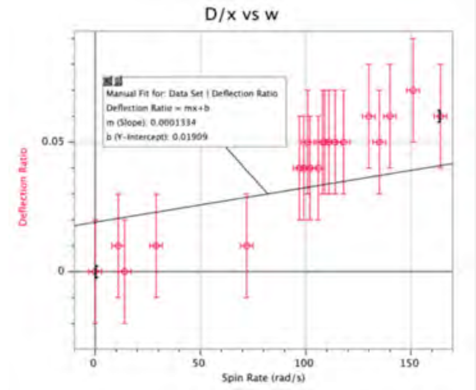

Figure 7: Linear graph of lowest possible slope.

Sample Calculations

10. Slope Uncertainty

$$Uncertainty = \frac{Maximum\ Value - Minimum\ Value}{2}$$

$$= \frac{0.0006834 - 0.0001334}{2}$$

$$= 0.000275$$

$$= \pm 0.0003$$

11. Y-intercept Uncertainty

$$Uncertainty = \frac{Maximum\ Value - Minimum\ Value}{2}$$

$$= \frac{0.01909 - (-0.01791)}{2}$$

$$= 0.0185$$

$$= \pm 0.02$$

Final Equation

$$\frac{\boldsymbol{D}}{\boldsymbol{x}} = (\mathbf{0.0004} \pm \mathbf{0.0003}\ \boldsymbol{rad^{-1}})\boldsymbol{\omega} + (\mathbf{0.00} \pm \mathbf{0.02}) \qquad \text{[Equation 3]}$$

Comment [9]: The report shows evidence of full and appropriate consideration of the impact of measurement uncertainty on the analysis.

Comment [10]: Appropriate and sufficient data processing is carried out with **the accuracy** required to enable a conclusion to the research question to be drawn that is fully **consistent** with the experimental data

12. Proportionality Constant (Based on theoretical equation from cited source)

$$Proportionality\ Constant = \frac{\pi R^3 \rho t}{m}$$

$$= \frac{\pi(0.02^3 m^3)(1.1841 kgm^{-3})(0.175s)}{0.00236}$$

$$= 0.002207\ s$$

13. Uncertainty of Proportionality Constant

$$Uncertainty = \frac{Maximum\ Value - Minimum\ Value}{2}$$

$$= \frac{\frac{\pi(0.0201^3 m^3)(1.1881 kgm^{-3})(0.176s)}{0.00235} - \frac{\pi(0.0199^3 m^3)(1.1802 kgm^{-3})(0.174s)}{0.00237}}{2}$$

$$= 0.000062 \quad = \pm 0.00006$$

Final Value of Theoretical Proportionality Constant

$$0.00221 \pm 0.00006$$

Interpretation of the Results

The results of the data processing show that there is a proportional relationship between the spin rate and the deflection ratio. The proportionality constant between the spin rate and the deflection ratio is experimentally shown to be 0.0004rad^{-1}, meaning the deflection ratio is 0.0004 times the spin rate in radians. The uncertainty in the proportionality constant (0.0003) is roughly 75% of the estimated value, implying high levels of uncertainty in the equation derived to model the situation.

Comment [11]: The processed data is correctly interpreted so that a completely valid and detailed conclusion to the research question can be deduced.

EVALUATION

Conclusion

From the results of the experiment and the analyzed data, it was evident that as the value of the Ping-Pong ball's spin rate increased, its deflection ratio increased as well. Clearly, the spin rate was directly proportional to the deflection ratio and in the end the following equation was derived to model the relationship between the two variables.

$$\frac{D}{x} = (0.0004 \pm 0.0003\ s)\omega + (0.00 \pm 0.02) \qquad \text{[Equation 4]}$$

The final graph was a proportional fit starting from the origin and steadily increasing in y-value, the deflection ratio, as the x-value, the spin rate, increased. This shows that the spin rate and deflection ratio are proportional to each other. As seen in the final graph, the fit managed to stay relatively close to all of the data points and within the uncertainty bars; therefore, a sufficient amount of confidence can be put onto the analyzed data points. Only about one or two data points didn't fit the proportional graph as well as the others did, which is most likely a bad data point possibly resulting from a bad hit. The uncertainties were pretty large due to the fact that the video analysis process was extremely sensitive. Although the same video was analyzed three times in a row, a slight difference in tracking the ball made a large difference. Despite the uncertainty, the data points collected in this experiment seemed to be accurate since they managed to keep true to the predicted type of graph and a final equation was able to be found from the relationship.

Comment [12]: A detailed conclusion is **described and justified** which is entirely relevant to the research question and fully supported by the data presented.

However, the slope of the graph did not match the calculated value. In fact, the calculated value was greater by roughly a factor of five, which suggests that there was either a flaw within the collected data or the original equation that was used to model the situation (Equation 1). Since the collected data seemed accurate to a decent extent, the equations used were analyzed. One assumption that Equation 1 had made was the fact that the surface of the ball had no effect on the ball as it spun through the air. However, the definition of Magnus Force, the force causing the curvature of the ball, states that there is a coefficient of air resistance, or more commonly known as the coefficient of drag, associated with it, which clearly shows that some constant was missing from the original equation. The assumption that the coefficient of drag was simply a value of one was refuted by the results of this investigation. Since drag coefficients are a value between zero and one, and the slope of the graph was $1/5^{th}$ of the calculated slope value, it can be predicted that the missing constant variable was what caused the calculated slope value to end up being to high. A constant k was inserted into derivation process and it showed that adding the constant along with the slope didn't disrupt the nature of the equation in representing the phenomenon. Therefore, the following equation would be a more accurate representation of the situation:

$$\frac{D}{x} = \frac{\pi R^3 \rho k t}{m}\omega \qquad \text{[Equation 5]}$$

where k is some constant that is acting on the ball during its flight. Nevertheless, it can be concluded that the spin rate of a Ping-Pong ball is directly proportional to the deflection ratio.

Comment [13]: A conclusion is correctly **described and justified** through relevant comparison to the accepted scientific context

Despite the differences in the slope values, it can be seen that the relationship between the two variables is proportional and this fact can be applied to spherical objects, most likely spherical sports balls, that gains enough air time to experience curvature during its flight. Although the slope values may differ, the general shape and nature of the graphs will be similar. For objects with small mass and size, and a smooth surface like that of Ping-Pong balls may share a similar slope value as well. The deflection ratio of other types of balls, such as golf balls, tennis balls, or baseballs, would increase with added spin, but the type of graph achieved is unknown since the surface of the ball may cause the final graph to be an increasing graph possibly linear, proportional, or even exponential. Although the proportionality of the two variables can be applied to a wide range of spherical objects, the specific type of graph that would be created cannot be.

Evaluation

One weakness of the experiment was the fact that the camera's view wasn't exactly perpendicular to the hit since it was first connected to a tripod, which was clamped onto a shelf near the ceiling. The camera ended up being slightly crooked and the angle of its view was being affected by a small amount every time the record button was pressed when starting and stopping the video recordings. This would also be a random error since the angle in which the cameras are recording could affect the distances being analyzed. In order to keep the camera's view as constant and perpendicular to the field of hit as possible, a different tripod can be used. A larger tripod that would allow the camera to be bend at a 180 degree angle looking straight down would work much better since the tripod can be raised to its maximum height and placed onto a table, the camera can be put facing down, and the balls can be hit right under the table while they are being recorded. This adjustment would allow greater consistency and accuracy of the data points.

Another weakness is the fact that the ball's distance from the camera couldn't be controlled. A ruler was set up horizontally using a retort stand in the field of the hit in order to act as a reference guide to keep all the hits roughly around the same height; however, the positions were all off by a slight factor. This would cause the collected data to have random errors since the curvature of the balls that were hit further away from the camera lens would seem to curve less than if it had been closer to the camera due to the difference in distance. This was an inconsistency within all the trials and affected the distance measurements to be slightly inaccurate. A related, and very important issue, is the fact that the ball was moving in a vertical parabola while traveling across the camera view, and the vertical motion was different every time, coming closer or farther from the camera as it moved across, depending on angle and speed of hit. In order to decrease the level of inaccuracy and maintain the height of the ball as close to each other as possible, a wooden board can be set up horizontally instead of a ruler as this would act as a better guide in knowing where to hit the ball. With a ruler, the height was approximated but with a wooden board or a flat plane, it would be much easier to keep the height of the ball consistent, which would decrease the inaccuracies in the data.

A third weakness in the experiment was the fact the ball's path range was extremely limited. The high-speed camera only was able to detect a narrow pathway since the distance between the ball and the camera was limited by the height of the room. This only allowed minor curvatures to be recorded because optimum spin was only achievable if the ball was hit for a longer time period, as Ping-Pong balls tend to curve most during the second half of its flight, not the first. Due to the limited space and low camera resolution at high-speed settings, only the first half of the ball's flight was able to be detected which overall resulted in minor spin. In order to obtain trials with greater spin, an abundant amount of space is needed since this would allow the camera to be positioned at a higher spot and the last bits of the ball's curvature would be able to be seen as well, which would result in greater deflection ratios. This would ultimately allow a great range of data and even more accurate equations and graphs would be achieved.

Comment [14]: Strengths and weaknesses of the investigation, such as limitations of the data and sources of error, are **discussed** and provide evidence of a clear understanding of the **methodological issues** involved in establishing the conclusion.

The student has **discussed** realistic and relevant suggestions for the improvement and extension of the investigation.

Comment [15]: The presentation of the investigation is clear. Any errors do not hamper understanding of the focus, process and outcomes.

The report is well structured and clear: the necessary information on focus, process and outcomes is present and presented in a coherent way.

The report is relevant and concise thereby facilitating a ready understanding of the focus, process and outcomes of the investigation.

The use of subject specific terminology and conventions is appropriate and correct. Any errors do not hamper understanding.