# INVESTIGATION OF FALLING BALL VISCOMETRY AND ITS ACCURACY GROUP R1

Evelyn Chou, Julia Glaser, Bella Goyal, Sherri Wykosky

### **ABSTRACT:**

A falling ball viscometer and its associated equations were studied in this experiment using sucrose solutions. Theoretical velocities at which the ball should fall were calculated by determining the drag coefficient corresponding to the Reynolds number for each solution. Theoretical and measured velocities were compared to assess the accuracy of the falling ball viscometer. Almost all of the experimental velocities were lower than theoretical velocities, with errors ranging from 27.53% error for the smallest ball to 47.05% error for the largest ball. Errors were due mostly to inaccuracies in timing, but also because the equations used to calculate theoretical velocity assumed the ball is falling in an infinitely extended liquid, whereas the ball was actually falling in an enclosed cylinder. The same falling ball viscometer was also investigated using a 1% gum guar solution. Through similar calculations as with the sucrose solution, it was found that gum guar has a low Reynolds number, and thus Stokes law applies to the gum guar solution. Application of Stokes law can be used to determine Newtonian properties of a fluid, and it was found that the gum guar solution is non-Newtonian. The hypothesis that values of the falling ball viscometer would provide accurate terminal velocities was disproved, but the hypothesis that the guar gum would behave according to Stokes' Law was verified.

#### **BACKGROUND:**

A falling ball viscometer consists of a cylinder filled with the fluid in question into which a ball of known density and dimensions is dropped. The velocity was calculated from the time it took a falling ball to pass a set distance between two markers on the graduated cylinder. The ball is assumed to reach terminal velocity over the timed distance. Comparing terminal velocities from different sized balls yields information about whether the fluid is Newtonian (ideal) or non-Newtonian. The viscosity of the solution can also be determined with this method. In this experiment, the fundamental physics behind the falling ball viscometer were used to determine its accuracy. The viscometer is meant to approximate an infinitely long column of infinite diameter.

The forces acting on a sphere falling in a falling ball viscometer consist of force due to gravity  $(F_g)$ , force due to the buoyancy of the solution  $(F_b)$ , and force due to drag on the ball from the liquid  $(F_d)$ . The forces can be summed as follows:

## $\mathbf{F}_{\mathrm{T}} = \mathbf{F}_{\mathrm{g}} - \mathbf{F}_{\mathrm{d}} - \mathbf{F}_{\mathrm{b}}$ [Equation 1]

Where  $F_T$  is total force. In a viscometer with a Newtonian fluid, the sphere is assumed to reach terminal velocity, at which  $F_T = 0$ .

#### $0 = F_g - F_d - F_b \qquad [Equation 2]$

Force due to gravity is equal to mass (m) times gravity(g). Mass of the sphere can be obtained by multiplying the density of the sphere ( $\rho_b$ ) by the volume (V). For a sphere, where r is the radius of the sphere,

$V = (4/3)*\pi r^{3}$	[Equation 3]
$m = (4/3)*\pi r^3 \rho_b$	[Equation 4]
$F_g = g^*(4/3)^* \pi^* r^{3*} \rho_b$	[Equation 5]

Force due to drag in a falling ball viscometer is a function of the coefficient of drag( $C_d$ ), velocity(v), density of the solution( $\rho_s$ ), and projected area (A). For a sphere, projected area is equal to the cross-sectional area of the sphere. The function is as follows:

$$F_d = 0.5 * C_d * \rho_s * v^2 * \pi * r^2$$
 [Equation 6]

Force due to buoyancy is a property of the solution and the volume of the ball. The formula for the volume of the sphere appears in Equation 3.

$$F_b = \rho_s * (4/3) * \pi * r^3 * g$$
 [Equation 7]

Combing Equations 5, 6, and 7 as per Equation 2 yields the sum of the forces acting on the ball when it is falling at terminal velocity.

$$0 = g^{*}(4/3)^{*}\pi^{*}r^{3*}\rho_{b} - 0.5^{*}C_{d}^{*}\rho_{s}^{*}v^{2*}\pi^{*}r^{2} - \rho_{s}^{*}(4/3)^{*}\pi^{*}r^{3*}g$$
 [Equation 8]

Rearranging and combining terms yields a relationship that can be used to calculate a theoretical velocity.

$$v^{2} = \frac{8*g*r(\rho_{b} - \rho_{s})}{3*C_{d}*\rho_{s}}$$
 [Equation 9]

Alternatively, the equation can be rearranged so that solving for a drag coefficient from a given velocity could be performed.

$$C_{d} = \frac{8*g*r(\rho_{b} - \rho_{s})}{3*v^{2}*\rho_{s}}$$
 [Equation 10]

For the sucrose experiments, theoretical velocities were calculated using Equation 9.  $C_d$  was found using a graph of drag coefficient versus Reynolds number for spheres<sup>1</sup> (see Figure 1 in Appendix), which was digitized by Dr. Foster so that it could be read more easily. Reynolds Number (N<sub>Re</sub>) was calculated as

$$N_{Re} = \frac{2*\rho_s * r^* v}{\eta}$$
 [Equation 11]

<sup>&</sup>lt;sup>1</sup> Figure 12.4 Drag coefficient versus Reynolds number for various objects, page 168. <u>Fundamentals of Momentum</u>, <u>Heat, and Mass Transfer</u>. Welty, Wicks, and Wilson. 3<sup>rd</sup> Edition. John Wiley and Sons, New York. 1984.

where  $\eta$  is dynamic viscosity and all other variables are as previously defined. Drag coefficient was then used in Equation 9 to determine a calculated velocity.

For the guar gum experiments, terminal velocity was known, and Equation 10 was used to determine the drag coefficient. Using the same graph of  $C_d$  vs  $N_{Re}$ , the  $N_{Re}$  of the guar gum solution was found. In the initial experiment performed with the guar gum, a formula derived from Stokes Law was used to determine the apparent viscosity of the guar gum.

$$v = [g(\rho_b - \rho_s) (2*r)^2]/18 \eta$$
 [Equation 12]

where all variables are as defined previously. Stokes' Law only holds for Reynolds numbers that are less than 10. The Reynolds numbers of the guar gum solution from the data recorded previously were determined to verify that Stokes Law was used appropriately.

# **METHODS:**

1500 mL of solutions that were 0 (water), 10, 20, 30, 40, and 50% sucrose by weight were prepared by massing the appropriate amounts of white table sugar and de-ionized water and mixing to complete dissolution of the sucrose. Viscosities and densities of the solutions were calculated using an online sucrose calculator<sup>2</sup> and appear in Table 1.

% Sucrose Solution	Density (kg/m <sup>3</sup> )	Viscosity (Pa*s)
0	996.5114	0.00082
10	997.1647	0.000841
20	1064.882	0.001499
30	1093.120	0.001962
40	1118.521	0.002592
50	1141.302	0.003397

 Table 1. Properties of Sucrose Solution with Increasing Concentration

The solutions were then poured into a 1000mL graduated cylinder and allowed to sit so that bubbles from stirring and pouring would come out of solution. The temperature of each solution was also taken.

Five balls of four different radii were released from the surface of the liquid and allowed to free fall. The balls were plastic with a density of 1400 kg/m<sup>3</sup> and had radii of 3.1mm, 4.7mm, 6.3mm and 9.45mm. Two group members timed the fall between two set points denoted by rubber bands on the outside of the cylinder. The first rubber band appeared well below the surface of the fluid so that it could be assumed that the falling balls reached terminal velocity. The distance between the two rubber bands was measured, and using the time and distance, velocity was calculated. The balls were subsequently retrieved from the graduated cylinder and the

<sup>&</sup>lt;sup>2</sup> http://www.seas.upenn.edu/courses/belab/ReferenceFiles/Viscometer/SucroseCalculator.html

experiment was repeated with 0% (water) and 10% sucrose and a marble of density 2517.8 kg/m<sup>3</sup> and radius of 8.555mm.

The experiment had been performed earlier in the semester using 1% guar gum as the solution. The balls used in that experiment were the same as those used for sucrose, and the experimental procedure was identical except with 1% guar gum instead of sucrose solutions.

% Sucrose Solution	Sphere Radius (m)	N <sub>Re</sub>	C <sub>d</sub>	Theoretical Velocity (m/s)	Experimental Velocity (m/s)
	0.0031	1869.967	0.384	0.292272	0.248275
00%	0.0047	3109.678	0.373	0.365146	0.272319
070	0.0063	4617.245	0.367	0.426196	0.301650
	0.00945	7328.563	0.381	0.512301	0.319189
	0.0031	1702.612	0.39	0.289685	0.231608
100%	0.0047	2698.181	0.375	0.363757	0.242087
1070	0.0063	3847.842	0.372	0.422840	0.257558
	0.00945	6376.777	0.379	0.513067	0.284557
	0.0031	1312.885	0.401	0.252147	0.196607
200%	0.0047	1566.347	0.393	0.313616	0.234563
20%	0.0063	1946.019	0.383	0.367804	0.217409
	0.00945	3455.154	0.372	0.457078	0.257339
	0.0031	588.834	0.48	0.217675	0.170472
200%	0.0047	1017.702	0.396	0.295086	0.196422
3070	0.0063	1558.238	0.391	0.343819	0.221981
	0.00945	2406.448	0.378	0.428270	0.228543
	0.0031	337.805	0.565	0.189957	0.126255
40%	0.0047	676.628	0.457	0.260069	0.166800
	0.0063	1043.777	0.387	0.327120	0.191960
	0.00945	1573.035	0.393	0.397666	0.192864
	0.00310	149.038	0.798	0.151696	0.071557
50 %	0.00470	344.417	0.559	0.223170	0.109070
30 %	0.00630	581.424	0.480	0.278832	0.137363
	0.00945	983.081	0.407	0.370862	0.154837

# **RESULTS:**

Table 2. Individual Falling Ball Data for Varying Sucrose Solutions

All experimental velocities were found to be statistically different from their theoretical counterparts. (p<0.05, n=10)

Ball Size	<b>Percent Error</b>	
Radius (m)	(%)	
Small (0.0031)	27.53	
Medium 1 (0.0047)	34.08	
Medium 2 (0.0063)	39.45	
Large (0.00945)	47.05	

 

 Table 3. Percent Error For Experimental Terminal Velocity as Compared to Actual Terminal Velocity Across All Sucrose Solutions

A general trend across each individual sucrose solution was observed that as the falling ball diameter increased the percent error also increased. As the falling ball diameter increased, the velocity increased, and the standard deviation of time that it took to fall increased.

% Sugaraga	Sphere Boding	Solution Vigoasity	Solution			Theoretical	Experimental
solution	(m)	(Pa*s)	$(kg/m^3)$	N <sub>Re</sub>	Cd	Velocity (m/s)	(m/s)
0 %	0.008555	0.00082	996.5114	28697.34	0.449	0.871864	1.380645
10%	0.008555	0.00084	997.1647	33113.85	0.469	0.852609	1.632258

Table 4. Fall Marble Data for Varying Sucrose Solutions

The density of the marble was  $2517.8 \text{ kg/m}^3$ . The percent error of the experimental terminal velocity as compared to the theoretical terminal velocity was 58.36% (0% solution), and 91.44% (10% solution). It should also be noted that the marble fell at an exceeding high velocity, making timing very difficult.

Radius	Terminal	Solution Viscosity		
(m)	Velocity (m/s)	(Pa*s)	N <sub>Re</sub>	C <sub>d</sub>
0.0031	0.00967	0.107237	0.559	346.645
0.0047	0.01522	0.212915	0.672	212.059
0.0063	0.02244	0.331804	0.852	130.826
0.00945	0.02994	0.611174	0.926	110.169

Table 5. Individual Falling Ball Properties When Dropped in 1% Guar Gum Solution

It should be noted that the density of the 1% Guar Gum solution was  $1000 \text{ kg/m}^3$ .

# **ANALYSIS:**

In investigating the falling ball viscometer, it was hypothesized that the experimental terminal velocity would be the same as the calculated terminal velocity. This hypothesis failed to be true. T-testing showed all experimental velocities were significantly different from theoretical velocities. The experimental terminal velocity was consistently lower than the calculated terminal velocity. A general trend across each individual sucrose solution was observed that as the ball diameter increased the percent error in terminal velocity also increased. As the size of the ball increased, the terminal velocity also increased. Velocity is distance traveled over time. In this experiment, the distance the ball fell remained constant, and the time it took the ball to fall changed between trials. The standard deviation for the fall times for each ball size increased with the size of the ball. This is due to the constraints of the stopwatch timer used and how quickly the timers response was to the ball falling. Faster moving balls are more difficult to time accurately; the fastest moving balls fell through the cylinder in under one second. The fastest experimental terminal velocity found was 0.319189 m/s, and the highest calculated terminal velocity was 0.513067 m/s. It is believed that the high error was largely due to the discrepancy between the speed at which the ball fell and the reflexes of our timers.

It was observed that the denser the ball, the faster the terminal velocity. However, unlike the plastic falling balls, the trials with the marble showed that the experimental terminal velocity was faster than the calculated terminal velocity. There was still a large percent error between the two values, and it is also believed to be because of the constraints of accurate timing.

In this experiment, it was assumed that the spheres were dropped in an infinite fluid with no boundary effects. The experimental conditions were far from the assumed ideal situation and the wall effects from the graduated cylinder were likely to have a non-negligible effect on the terminal velocity. The walls of the graduated cylinder would affect the natural flow pattern of the fluid around the sphere, thus decreasing the terminal velocity. The experimental velocities were in fact, less than theoretical velocities. The corrected terminal velocity as derived by Faxen (1923) involves a power series expansion as follows:

$$v_t$$
(corrected) =  $v[1 - 2.104(d/D) + 2.09(d/D)^3 - 0.95(d/D)^5 + ...]^{-1}$  [Equation 13]

where  $v_t$  is the terminal velocity correct for the wall effects, d is the diameter of the dropped sphere, and D is the diameter of the graduated cylinder<sup>3</sup>. In future experiments, this should be considered to improve accuracy.

In the guar gum viscosity experiment performed during the lab rotation, the goal was to determine that guar gum is a non-Newtonian fluid by applying Stokes Law. Equation 12 was rearranged to find the ratio  $v/d^2$ , where v is the terminal velocity and d is the ball diameter. It was found in the previous lab that the ratio  $v/d^2$  was not constant with varying ball sizes, as it should be for a Newtonian fluid. It was prematurely concluded that the guar gum solution is non-Newtonian due to the conclusion drawn from finding that  $v/d^2$  was not a constant.

<sup>&</sup>lt;sup>3</sup> Flude, MJC and Daborn, JE. Viscosity measurement by means of falling spheres compared with capillary viscometry. Journal of Physics E: Scientific Instruments, Vol. 15, 1982.

The previous lab assumed that  $N_{Re}$  was equal to zero for Equation 12 to be applicable in Stokes Law. In this lab's investigation, through reverse engineering, the Reynolds number was found from Figure 1 by first calculating the drag coefficient from Equation 10. It was discovered that the Reynolds numbers for the guar gum trials were all less than 1 ( $N_{Re}$ <1) which is sufficient for the assumption that  $N_{Re}$ <10 for Stokes Law to be applicable. With the fulfilled assumption that Reynolds number has to be less than 10 in order for Stokes Law to be applied, Equation 12 is valid in the previous calculations for the v/d<sup>2</sup> ratio values. Thus, Stokes Law and Equation 12 are applicable and therefore the previous conclusion that guar gum solution is non-Newtonian (pseudoplastic) has been verified.

As an extension of this investigation, it would be beneficial to understand how the variable factor of temperature affects flow and terminal velocity. If at all possible, a taller and wider graduated cylinder should be used in order to better simulate an infinite cylinder with an infinite width—a more ideal flow. For the purposes of this investigation, the stopwatches used to measure the time of falling balls served the purpose of this investigation. However, as an improvement, or if this experiment were to be conducted within industry, a better system of timing should be devised in order to gain more accurate measurements of terminal velocities. A possible method to have more accurate time readings is through the use of position-dependent time sensors or perhaps a digital camera that records time in hundredths of seconds. These, as well as many other methods, would reduce, or even eliminate, the human error found in this investigation.

# **APPENDIX:**



Figure 1: Graph used to determine drag coefficient from Reynolds Number



Figure 2: Using the computer program CurveUnscan, a portion of the graph from Welty was digitized to give an Excel file of actual data points for Reynolds numbers and their corresponding drag coefficients. This minimized the error inherent in reading from the graph. The resultant graph, also generated from Excel, appears above.

% Sucrose Solution	Sphere Radius (m)	Theoretical Velocity (m/s)	Experimental Velocity (m/s)	t calc
0	0.0031	0.292272	0.248275	9.457621
	0.0047	0.365146	0.272319	13.94843
	0.0063	0.426196	0.30165	11.00245
	0.00945	0.512301	0.319189	34.64174
0.1	0.0031	0.289685	0.231608	7.381084
	0.0047	0.363757	0.242087	16.24119
	0.0063	0.42284	0.257558	24.74565
	0.00945	0.513067	0.284557	35.73196
0.2	0.0031	0.252147	0.196607	8.227567
	0.0047	0.313616	0.234563	14.65952
	0.0063	0.367804	0.217409	44.01441
	0.00945	0.457078	0.257339	28.93646
0.3	0.0031	0.217675	0.170472	29.23239
	0.0047	0.295086	0.196422	35.90512
	0.0063	0.343819	0.221981	37.65188
	0.00945	0.42827	0.228543	65.31413
0.4	0.0031	0.189957	0.126255	39.45032
	0.0047	0.260069	0.1668	33.94188
	0.0063	0.3272	0.19196	41.79362
	0.00945	0.397666	0.192864	66.97364
0.5	0.0031	0.151696	0.071557	165.0733
	0.0047	0.22317	0.10907	192.6083
	0.0063	0.278832	0.137363	133.1801
	0.00945	0.370862	0.154837	135.8608

Figure 3: t-values calculated using a confidence interval of t(crit) = 2.306 (df = 9) for the experimental vs. theoretical velocities