# Water Oscillation in an Open Ended Cylinder Tube

### Introduction

Simple harmonic motion is a motion of an object in which the periods and the amplitude of the motion are constant. An example of simple harmonic motion is oscillation of mass on a spring.



Fig 1: Simple Harmonic Motion

Damped simple harmonic motion occurs when there is constant force acting on the oscillation; e.g. friction or air resistance. Due to the constant force, the energy of the simple harmonic motion is gradually converted into heat; the period remains the same and the amplitude gradually decreases.



Fig 2: Damped Simple Harmonic Motion

The formula for the period of both simple harmonic and damped simple harmonic motion is the same. The formula for the period is...

$$T = 2\pi \sqrt{\frac{m}{k}}$$

... where m is the mass of the object oscillating and k is the spring constant (or force constant).

The motion of the water level of oscillating water in a tube is also a "near" damped simple harmonic motion. The following is the derivation for the period involved in a water oscillation in a tube.



Fig 3: Water oscillation in a tube

$$T = 2\pi \sqrt{\frac{m}{k}}$$

... where 
$$k$$
=force constant,  $m$ =mass of water,  $T$ =Period of Oscillation

$$F = ma$$
 ,  $F = -k(\Delta \mathbf{x})$ 

...where a=acceleration of water,  $\Delta x$ =displacement of water

$$ma = -k(\Delta \mathbf{x})$$
$$\frac{m}{k} = (-)\frac{\Delta \mathbf{x}}{a} \quad \dots \dots \quad (1)$$

 $m = lA\rho$ , where m=mass of water, A=cross sectional area,  $\rho$ =density of water, l=length of water column in tube

 $F = ma = lA\rho a$ ... Force for water indicated in blue  $F = \Delta x A \rho g$ ..... restoring force (g=gravity)

 $lA\rho a = \Delta x A \rho g \dots A$  and  $\rho$  cancel out

$$la = \Delta xg \dots$$
 By rearranging this  $\rightarrow \frac{\Delta x}{a} = \frac{l}{g} \dots 2$ 

Combining equation (1), (2)  $\frac{l}{g} = \frac{\Delta x}{a} = \frac{m}{k}$ ... Thus  $\frac{l}{g} = \frac{m}{k}$ .....(3)

If ③ is substituted to the period equation,  $\rightarrow T = 2\pi \sqrt{\frac{l}{g}}$ 

### **Research Question**

Is the oscillation of water in a tube an exact simple harmonic motion?



Fig 4: Damped simple Harmonic Motion in water tube

# Hypothesis

For large oscillations, the oscillation of water in a tube will have an inconsistent period, and therefore will not be an exact damped simple harmonic motion. For small oscillations, the periods will become similar.

There will be an end correction; the entire oscillation will involve an additional water column at the bottom of the tube, which would be proportional to the average height at which the water is oscillating at.

### Explanation

The equation ...

$$T = 2\pi \sqrt{\frac{l}{g}}$$

···indicates that that the period is directly related to the square root of height of the water level in the tube. Therefore, for large oscillations, where the change in *l* is large, there will also be a large change in the period. One could also say that as the change in *l* is large, the change in m would also be large;  $m = lA\rho$ . This, if applied to the equation ···

$$T = 2\pi \sqrt{\frac{m}{k}}$$

... explains why the period will be inconsistent. For small oscillations, the change in l or m becomes small, thus the periods will be similar.

The End Correction/ Additional water column under the tube will exist because the water tube has an open end; the motion of the water is not limited to within the length of the tube but it actually moves in and out between the tube and the water tank. The end correction would be proportional to the average height at which the water is oscillating at because the larger the mass of the water, the more it will push down; which would create a greater end correction.

### Variables

To Control:	<ul> <li>Dimension (width, length, height, radius) of water tank and tube</li> <li>Amount of water in the tank</li> </ul>
To Measure:	- Height of the water level in the tube in relation with time
To Change:	- Small oscillation/End correction Data collection: Depth to which the tube is put in to the water

### Procedure

#### -Large/Small Oscillation

A water tank with height, 0.815m, was filled with water. A transparent tube, open at both ends, was put into the water. To clearly observe the motion of the water level, a small piece of styrofoam was put into the tube to float on the water surface. The tube was held up vertically so that only 1/4 of its total length was under the water. With one hand blocking the top end of the tube, tube was pushed down in to the water. The hand was then released; the oscillation was recorded, using a camera. To get a clear film, black poster paper was attached to the back of the water tank and light was shone on the tank at an angle. This method above was repeated 3 times and each was analyzed using logger pro.

The procedure was repeated for oscillations in the opposite way; the tube was let to stand vertically in at the lowest possible point in the water tank. With one hand blocking the top end, the tube was dragged up to 3/4 of its total the length. The hand was then released and the motion of the water level was recorded on the camera. This method was repeated 3 times and each was analyzed using logger pro.

### -End Correction Data collection

The previous procedure was repeated for small oscillations. A meter stick was attached to the side of the tube to indicate the how deep the tube went into the water. To create small oscillations, the initial displacement (length to which the tube was pushed down from the top) was 0.02m or less. The oscillation recordings were started out with the tube at shallower depths, 0.045m, to deeper depths, 0.465m. Each motion was analyzed on logger pro. The period of each motion was calculated by finding the time taken for 4-6 periods and dividing that by the number of periods accounted. Accounting more than 4 periods was done to increase accuracy. The end corrections were found using the data of the periods and the derivation in the following page.

# -End Correction Equation derivation



Fig 5: Water Tube and End Correction

Using the equation...

$$T = 2\pi \sqrt{\frac{l}{g}}$$

... and substituting

$$l = l_x + l_0$$

... where  $l_0$ =length of end correction,  $l_x$ =depth of tube under water

$$T = 2\pi \sqrt{\frac{l}{g}} \cdots T = 2\pi \sqrt{\frac{l_x + l_0}{g}} \cdots \frac{T}{2\pi} = \sqrt{\frac{l_x + l_0}{g}}$$
$$\left(\frac{T}{2\pi}\right)^2 = \frac{l_x + l_0}{g} \cdots g\left(\frac{T}{2\pi}\right)^2 = l_x + l_0$$
$$l_0 = g\left(\frac{T}{2\pi}\right)^2 - l_x$$

## Results

Data



Graph 1: Oscillation of Water in a tube

1/2 Periods	Time at point n+1 (s) ± 0.0005	Time at point n (s) $\pm 0.0005$	Difference in Time $n+1,n(s) \pm 0.001$	
1	1.295	0.446	0.849	
2	1.964	1.295	0.669	
3	2.679	1.964	0.715	
4	3.393	2.679	0.714	
5	4.152	3.393	0.759	
6	4.821	4.152	0.669	
7	5.714	4.821	0.893	
8	6.339	5.714	0.625	
9	7.071	6.339	0.732	
10	7.813	7.071	0.742	
11	8.536	7.813	0.723	
12	9.277	8.536	0.741	
13	10	9.277	0.723	

Table 1: Inconsistent 1/2 periods for large oscillations

# Small Oscillation/ End Correction

# Sample calculation

Small oscillation when bottom of tube is 0.295m deep in water



Graph 2: Upper end of 4 periods

Graph 3: Lower end of 4 periods

 $\rightarrow$  Upper end of 4 periods- 48.52s ..... Lower end of 4 periods – 44.04s

→Period 
$$(T) = \frac{48.52 - 44.04}{4} = 1.12 \text{ s} \cdots$$
Plug in to equation  $l_0 = g \left(\frac{T}{2\pi}\right)^2 - l_x$ 

$$→ l_0 = g \left(\frac{1.12}{2\pi}\right)^2 - 0.295 \text{ (Use approximate values } → g = 9.8 \text{ m/s}, 2\pi = 6.28 \text{ )}$$
$$= 0.0167 \text{ m}$$

Error Calculations  

$$T \rightarrow \pm 0.0005 \dots l_x \rightarrow \pm 0.0005$$
  
 $\left\{ g \left( \frac{1.120 + 0.0005}{2\pi} \right)^2 - (0.295 - 0.0005) \right\} - \left\{ g \left( \frac{1.12}{2\pi} \right)^2 - 0.295 \right\} = \underline{0.000778m}$ 

Thus, error for end correction at depth 0.295m is  $\pm 0.000778m$ 

Data Set					
Length± 0.0005	Period ± 0.0005	End c.	Error		
(m)	(S)	(m)	(m)		
0.045	0.500	0.0171	0.000624		
0.107	0.707	0.0172	0.000676		
0.210	0.960	0.0190	0.000739		
0.255	1.050	0.0190	0.000761		
0.295	1.120	0.0167	0.000778		
0.365	1.240	0.0171	0.000808		
0.440	1.368	0.0250	0.000840		
0.465	1.396	0.0193	0.000847		

Table 2: End Correction



Graph 4: Length and End Correction



Graph 5: Period and End Correction

### **Data Analysis**

Graph 1 and Table 1 show that the  $\frac{1}{2}$  periods are inconsistent for large oscillations which are roughly for the first 4 periods. For the following periods, the  $\frac{1}{2}$  periods are constant.

The end correction for each points are scattered about the line y=0.01815m as shown in Graph 4; the points are scattered around y=0.01800m as shown in Graph 5. There is one anomaly when depth of tube is 0.445m; it gives an end correction value of 0.025m.

### Discussion

From Graph 1 and Table 1, it is clear that the period is inconsistent for large oscillations and relatively similar for small oscillations. From the first to the eighth 1/2period, time values are inconsistent; and from the 9<sup>th</sup> to the 13<sup>th</sup> 1/2 periods, the time values are consistent.

Graph 4 and Table 2 show that there must be an end correction to the water oscillation. However, the graph and the table shows that the end correction is not "proportional" to how deep the tube is in the water, but is approximately the same value for any depth; thus the end correction is independent of the mass involved in the water oscillation.

Another way to prove that there must be an end correction is by plugging in the

period value into the equation  $g\left(\frac{T}{2\pi}\right)^2 = l$ . The resulting value *l* always comes at larger

than its respective depth of the tube in the water.

### **Evaluation + Suggestion for further work**

The results of the experiment were very accurate; the percentage errors for the calculations of half periods were within 1%; for the end corrections, within 4.5%. The data also corresponded to what was expected. The water oscillation isn't a damped simple harmonic motion. It only approaches a damped simple harmonic motion, when the oscillations are small. This supports the hypothesis earlier stated.

The end correction analysis also supports the hypothesis about the end correction. However, it was found out that the end correction is not proportional to how deep the tube is put into the water but independent of the depth. It is the possibility that the end correction depends on the width of the tube, the dimension of the water tank or the total amount of water in the tank. With such variables, extended experimentation on the end correction of a water oscillation could be made.

There is room for improvements also within this experiment for greater accuracy. In this experiment, we assumed that the water oscillation only involves the motion in the tube, and that the water level of the tank stays constant. Actually, as the water level in the tube goes up, the water level of the tank goes down, and vice versa. The two motions are related. To make the assumption more close to reality, a tank with a longer width and length must be used, so that the oscillation of water level in the tank becomes insignificant.

Also holding the tube by hand posed several problems of error and inaccuracy. The tube wasn't always in a fixed position, or sometimes not exactly vertical. These factors could have caused anomalies like the end correction result for depth, 0.440m. This problem could be solved by using a large stand. Firstly, the air could be pushed down using the hand; then the tube can be held using the stand, and be kept in a vertically fixed position.

### References

http://hyperphysics.phy-astr.gsu.edu/hbase/shm.html

http://www.physics.uoguelph.ca/tutorials/shm/Q.shm.html